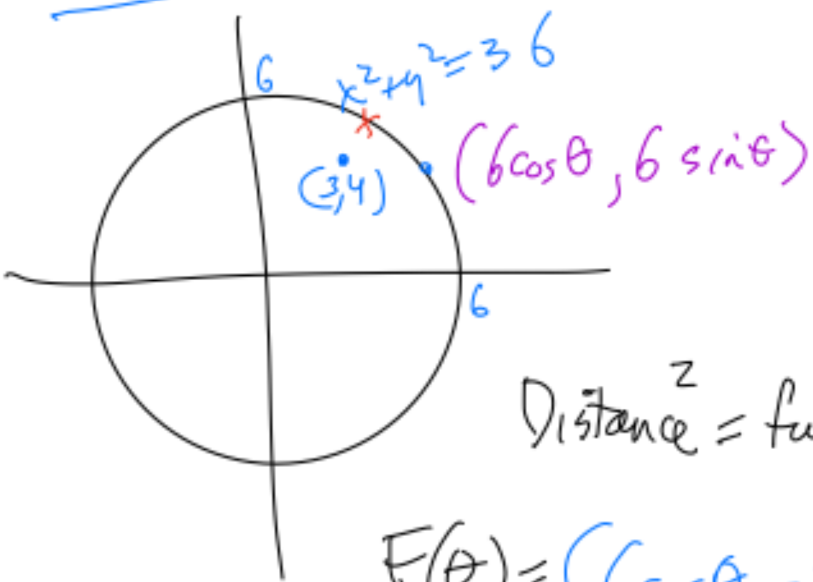
Find the point on the graph of $x^2+y^2=36$ that is closest to $(3,4)$.

Method 2.



$\text{Distance}^2 = \text{function } F(\theta)$

$$F(\theta) = (6\cos\theta - 3)^2 + (6\sin\theta - 4)^2$$

$$0 \le \theta \le \frac{\pi}{2}$$

Closed interval: possible mins are endpoints ($\theta = 0$, $\theta = \pi/2$) or or pt on inside:

$$F'(\theta) = 2(6\cos\theta - 3)(-6\sin\theta) + 2(6\sin\theta - 4)(6\cos\theta) = 0$$

divide by ~~6~~ 12 $\Rightarrow$

$$-\sin\theta(6\cos\theta - 3) + \cos\theta(6\sin\theta - 4) = 0$$

$$\Rightarrow -\cancel{6\sin\theta\cos\theta} + 3\sin\theta + \cancel{6\sin\theta\cos\theta} - 4\cos\theta = 0$$

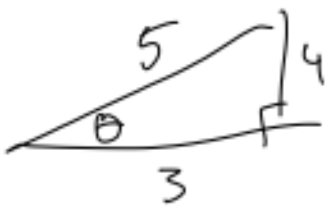
$$\Rightarrow \quad 3\sin\theta - 4\cos\theta = 0$$

$$3\sin\theta = 4\cos\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{4}{3}$$

$$\theta = \arctan\left(\frac{4}{3}\right).$$

$$(x,y) = \Big(6\cos\big(\underbrace{\arctan\left(\tfrac{4}{3}\right)}_{\theta}\big),\ 6\sin\big(\underbrace{\arctan\left(\tfrac{4}{3}\right)}_{\theta}\big)\Big)$$



$$\Rightarrow \cos\theta = \frac{3}{5}$$
$$\sin\theta = \frac{4}{5}$$

$$(x,y) = \left(6\left(\frac{3}{5}\right),\ 6\left(\frac{4}{5}\right)\right)$$

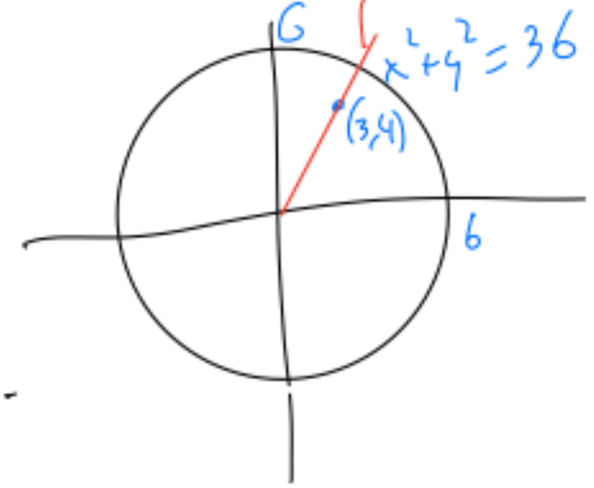
$$= \left(\frac{18}{5}, \frac{24}{5}\right) = (3.6, 4.8)$$

θ or (x,y)	distance
$\left(\frac{18}{5}, \frac{24}{5}\right)$	1 ← min value
0	} larger #s
$\pi/2$	

$\boxed{\therefore\ (3.6, 4.8) \text{ is the closest pt.}}$

Find the point on the graph of $x^2+y^2=36$ that is closest to $(3,4)$.

Method 3



$y=\frac{4}{3}x$ intersect with $x^2+y^2=36$

$$\Rightarrow \quad x^2+\left(\frac{4}{3}x\right)^2=36$$

$$x^2+\frac{16}{9}x^2=36$$

$$\frac{25}{9}x^2=36$$

$$x^2=\frac{36\cdot 9}{25}$$

$$x^2=\frac{(18)^2}{(5)^2}$$

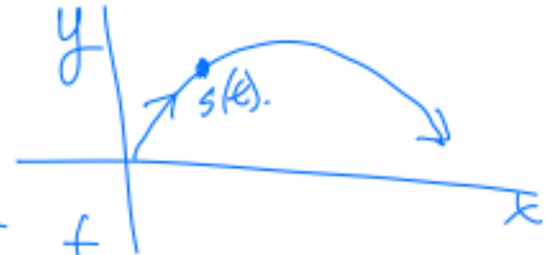
$$\Rightarrow \quad x=\frac{18}{5}$$

$$y=\frac{4}{3}\cdot\frac{18}{5}=\frac{24}{5}$$

$$\boxed{(x,y)=\left(\frac{18}{5},\frac{24}{5}\right)}$$

No calculator ☹

Projectile Motion



$S(t)$ = position at time t

$s(t) = (x(t), y(t))$

velocity $v(t) = s'(t) = (x'(t), y'(t))$

acceleration $a(t) = v'(t) = s''(t) = (x''(t), y''(t))$

Always.

Projectile motion: Acceleration due to gravity

$= \text{constant} = \boxed{-9.8\ m/s^2 = -32\ ft/s^2}$ downward ($y''(t)$) downward

$\Rightarrow x''(t) = 0$

$x'(t) = \text{constant} = V_{0x}$ = initial x velocity
(m/s or ft/s)

$x(t) = 5t + (\text{bubba})$

37.8
"
initial x position
$t=0 \quad x = \text{bubba}$

$\Rightarrow X(t) = (V_{0x})t + X_0$ initial x.

y - direction (vertical)

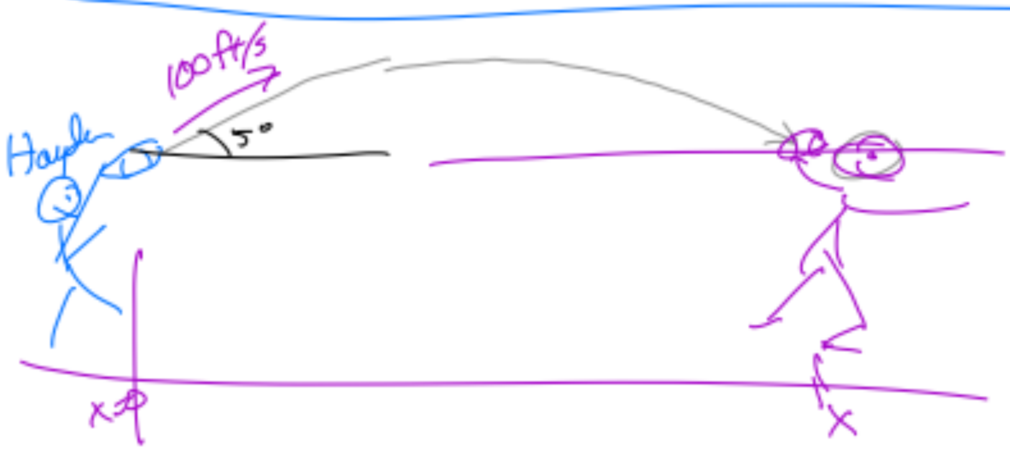
$$a(t) = y''(t) = -32\,\text{ft}/\text{sec}^2 = -9.8\,\frac{m}{s^2}$$

$$y'(t) = \underset{(-9.8)_{\text{metric}}}{-32}\,t + V_{0y} \leftarrow \text{initial velocity.}$$

$$\Rightarrow y(t) = \underset{(-4.9)_{\text{metric}}}{-16}\,t^2 + V_{0y}\,t + y_0$$

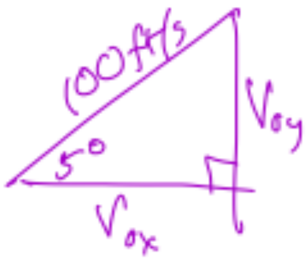
y_0 ← initial y-position

Example ① If Hayden throws a football to Cen using an initial velocity of 100 ft/s and angle 5° above the horizontal, and Cen catches it perfectly, how far away was Cen?



$$X_0 = x(0) = 0 \qquad t = 0$$

$$x(t) = (V_{0x})t + X_0 = (V_{0x})t$$



$$V_{ox} = 100(\cos(5°)) = 99.62 \text{ ft/s}$$

$$V_{oy} = 100(\sin 5°) = 8.72 \text{ ft/s}$$

$$x(t) = (99.62)t \qquad (\text{ft}) \qquad t \text{ in seconds.}$$

$$y(t) = -16t^2 + V_{oy}t + y_0$$

$$y(t) = -16t^2 + 8.72t + 0$$

Want $y(\text{last time}) = 0$

when Cen catches.